

Calculators and mobile phones are not allowed.

Answer all of the following questions.

1. [6 points each] Evaluate the following integrals:

(a)

$$\int \frac{\ln 2x}{x^2} dx$$

(b)

$$\int \frac{dx}{x^2 \sqrt{x^2 + 4}}$$

(c)

$$\int \frac{2x^2}{(x^2 + 1)(x - 1)} dx$$

(d)

$$\int \sqrt{1 + \sqrt[3]{x}} dx$$

(e)

$$\int \frac{\cot^3 x}{\sin^{\frac{3}{2}} x} dx$$

2. [6 points] Determine whether the following integral converges, if it converges, find its value.

$$\int_{-2}^2 \frac{dx}{(1-x)^{\frac{2}{3}}}$$

3. [3+4 points]

(a) Let C be the curve given by the parametric equations

$$x = 2 \cot t, \quad y = 2 \sin^2 t; \quad \pi/4 \leq t \leq 3\pi/4.$$

Find the point where the tangent line to the curve C is horizontal.

(b) Find the slope of the tangent line to the graph of $r = e^{3\theta}$ at the point that corresponds to $\theta = \pi/2$.

4. [2+5 points] Let R be the region that is inside the graphs of both of the equations $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$.

(a) Sketch the region R .

(b) Find the area of the region R .